Title: Math and Art

Brief Overview:

Students will have an opportunity, in groups, to discover the mathematical relationship between the number of sides in a regular polygon and the measure of its interior angles. Students will learn the construction techniques of geometry and use these techniques to create art that resembles the art of various cultures.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
  Students will bring together geometric concepts and compass construction skills to create original art designs. They will work in pairs to discover a pattern to predict the measure of interior angles of any regular polygon. Students also will work as a class to develop a formula for determining the interior angle measure of any regular polygon. Group data will be collected and inferential reasoning used. Finally, they will explore the applications of math in art and gain an understanding of the underlying geometric concepts that tie our multi-cultural world together.

- **Patterns, Functions, and Algebra**
  Students will find geometric patterns in nature and in the art of many cultures. They will develop a formula for determining the interior angle measure of any regular polygon

- **Geometry and Spatial Sense**
  Students will demonstrate an ability to construct line segments, angles, bisectors, regular polygons, and circle designs.

Links to Maryland High School Mathematics Core Learning Goals:

Functions and Algebra

- **1.1.1**
  Students will analyze patterns and extend them into a functional relationship.

- **1.1.2**
  Students will use tables, graphs (extension), and mathematical expressions to represent patterns and functional relationships

Geometry, Measurement, and Reasoning

- **2.1.1**
  Students will analyze and describe the characteristics of regular polygons and will construct same, as well as other geometric figures.

- **2.1.2**
  Students will identify and verify properties of geometric figures using concepts of algebra.

- **2.1.4**
  Students will validate the properties of geometric figures using appropriate tools and technology.

Grade/Level:

Grades 6 - 10

Duration/Length:

This activity will take 2 to 3 class periods (45-50 mins). Time will vary depending on class duration and student's prior knowledge

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Using a straight edge to draw a line
- Using a compass
- Calculating the mean
- Recognizing basic polygon shapes
- Classifying angles and angle measures

Student Outcomes:

Students will:

- work cooperatively in groups.
- collect and organize data.
- develop mathematical relationships from collected data.
- make connections between math and art.
- create art using mathematical constructions.

Materials/Resources/Printed Materials:

- Pencils, Colored pencils, Pens, Markers
- Paper
- Compass, Protractor
- Straightedge
- Eraser (large art style)
- Geometric tiles
- Student Worksheets 1 - 3

Development/Procedures:

DAY 1:

- Motivate the lesson with a discussion of the appearance of **implied** polygons in nature or in art. There are many examples, such as the star fish (pentagon), pine tree (triangle), or daisy (hexagon). Students can be asked to bring in objects that are good examples or find pictures in books that demonstrate the concepts. Project transparencies or shadows onto the board and allow student to find and trace the implied polygons .
- Discuss with the class the definition of a **regular** polygon. Reinforce the concept that **every triangle** has a sum of interior angles equal to 180°.
- Distribute one of the two versions of **Worksheet 1** to each group. Ask the groups to come up with a method for discovering the measure of the interior angles for each polygon on that group's sheet without using a protractor or any other measuring device. Have them record their observations and techniques on the worksheet.
- Call on groups, one at a time, to present the technique each has developed and the data collected. Be open to non-traditional methods of discovery but lead the class into dividing each polynomial into triangles. Have all students record every group's data.

- Give groups an opportunity to use class data to develop a formula for predicting interior angle measure for any regular polygon. Afterwards, call the groups together and have each present the results. Lead the discussion towards the conclusion that the number of sides can be used to predict the interior angle measure, using the mathematical formula $\frac{(n-2)\cdot 180^\circ}{n}$, where n = the number of sides in the polygon.

DAY 2:

- Give each student a compass and straightedge.
- Using a compass and a straightedge, demonstrate the constructions of a line segment, angle, segment bisector, and angle bisector. Methods for these constructions can be found in geometry text. Allow students ample time to practice each construction.
- Hand out **Worksheet 2** .
- Ask students to use the construction techniques to complete the indicated constructions. Students should be able to use these techniques to synthesize the construction of the regular polygons given as a combination of segments and angles.

DAY 3:

- Pass out compasses, straightedges, coloring tools, and paper.
- Give each student a copy of **Worksheet 3.**
- Discuss the information on the worksheet pertaining to mandalas. Ask students to think of places they may have seen examples of mandalas. What cultural influences may have prompted their use? Have students themselves ever used mandalas. HOW??
- Direct students in constructing mandalas using the worksheet as a guide.
- Allow students time to experiment and discover**.**

Performance Assessment:

- Have students share their results with each other.
- Assess daily progress through observation and questioning techniques.
- Collect worksheets each day.

General scoring rubric for the constructions on Worksheet 2:

4 - Good construction techniques with the use of a straightedge. Accurate representation of the segment or angle.

3 - Obvious construction technique, but less than accurate representation OR poor use of tools (straightedge/compass).

2 - Attempt at construction obvious, but poor representation AND poor use of tools.

1 - Obvious faked construction, poor representations, little or no apparent use of tools.

- Art work should be assessed with an eye toward the use of the techniques described in **Worksheet 3**. Allow for individual freedom of expression, but look for obvious use of construction techniques and tools.

Extension/Follow Up:

- Take students on a walking tour around your school. Have them look for and record any examples of polygons or mandalas they may find in nature, architecture, art displayed in your building, etc.
- Consider using calculator technology to gather the data from Day 1. Display in chart or table form. Perform a curve fit using a graphing calculator to find an equation relating the number of sides to total interior degrees of polygons.

- Extend construction lesson to include: perpendicular bisectors, dividing angles or line segments into powers of two through repeated bisection, parallel line segments, etc.
- Divide a circle per the suggestion on **Worksheet 3**. Create a different mandala by connecting every fourth division of the circle using only straight lines. Allow the student to experiment with other techniques to create individual signature pieces.

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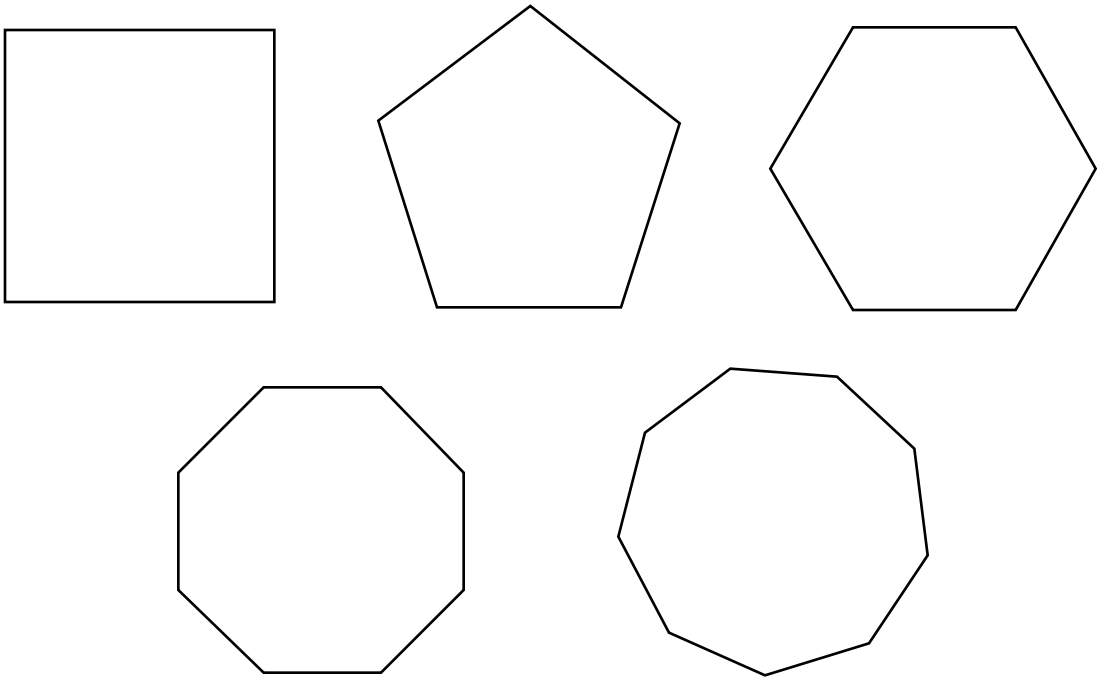
Worksheet 1-A

INTERIOR ANGLE MEASURE OF REGULAR POLYGONS

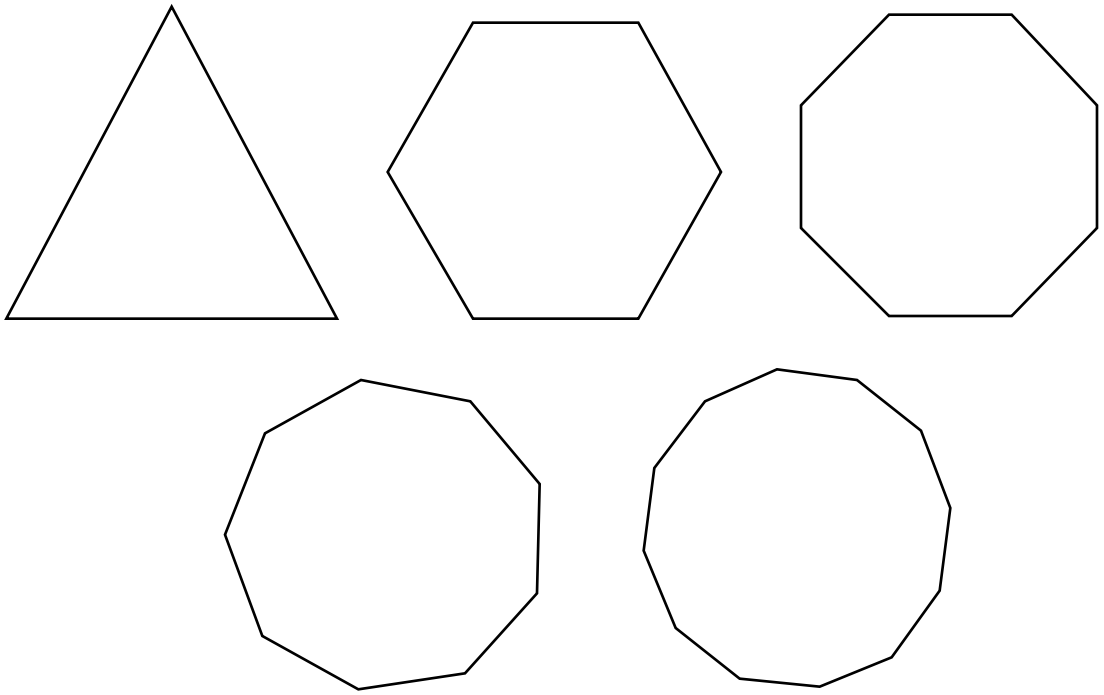


Name	Number of Sides	Total Degrees	Angle Measure

Describe below the technique your group used to determine the measure of the interior angle. BE SPECIFIC!!

Worksheet 1-B

INTERIOR ANGLE MEASURE OF REGULAR POLYGONS



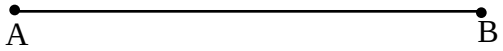
Name	Number of Sides	Total Degrees	Angle Measure

Describe below the technique your group used to determine the measure of the interior angle. BE SPECIFIC!!

Worksheet 2

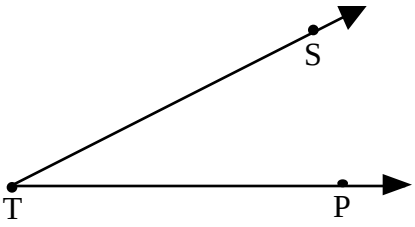
Constructions of Line Segments, Angles, Bisectors, and Polygons

1. Construct a line segment congruent to $\overline{AB}$

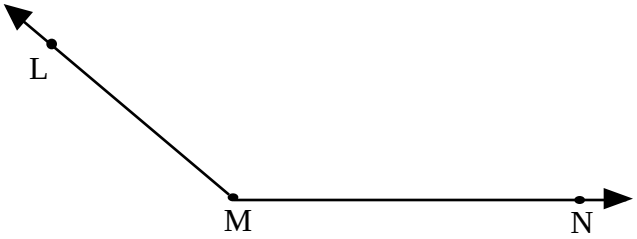


2. Construct a bisector of line segment $\overline{CD}$

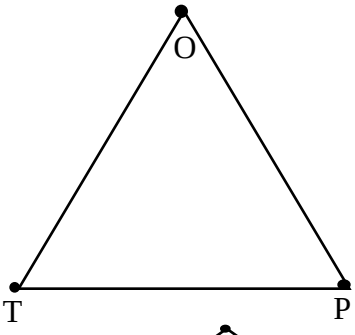
3. Construct an angle congruent to $\angle STP$



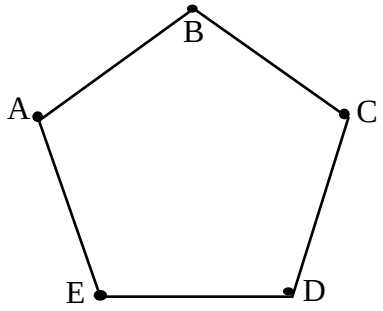
4. Construct an angle bisector for $\angle LMN$



5. Construct a triangle congruent to ΔTOP



6. Construct a pentagon congruent to $\pentagon ABCDE$



Worksheet 3

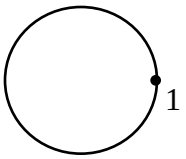
Art Construction

Mandalas

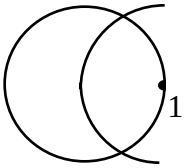
The word Mandala is from the Hindu Sanskrit language and means circle or center. Usually they are arranged in concentric layers. The Hindus used them for meditation. They appear in many other cultures as well. The Aztecs created their calendar in the form of a mandala. The Indians of the Southwest believed that multicolored sands arranged in a mandala enhanced healing rituals. Many churches have stained glass windows in a mandala pattern. Much of Islamic art uses mandalas that incorporate many geometric figures.

Here are some directions to help you create your own mandala design.

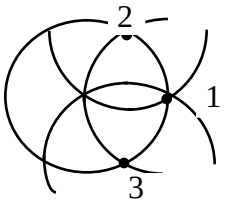
1. Using a compass, draw a circle of arbitrary radius. Place a point anywhere on its circumference.



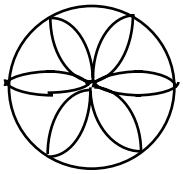
2. Using the same compass measurement, place the point of the compass on point 1. Swing an arc that intersects the circle twice and passes through its center.



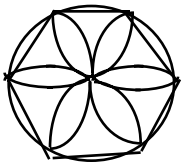
3. Repeat this process from each intersection point (points 2 and 3) being sure to maintain the same compass measurement throughout.



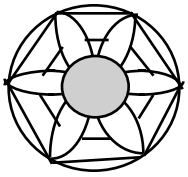
4. Continue with each new intersection point until you have six (6) arcs that form the daisy pattern at right. Erase any extraneous marks.



5. Connect the tips of the petals to form a regular hexagon.



6. Use additional geometric figures to personalize your mandala. Add color for the final touch.



Suggestion: Try creating a twelve pointed daisy pattern. How could you do this? (Hint: Think bisection !!)